The entanglement character between atoms in the non-degenerate two photons Tavis-Cummings model

Guo-feng Zhang * and Zi-yu Chen

Department of physics, School of sciences, Beijing University of Aeronautics and Astronautics (BUAA), Beijing 100083, China

Abstract

The entanglement character including the so-called sudden death effect between atoms in the non-degenerate two photons Tavis-Cummings model is studied by means of concurrence. The results show that the so-called sudden death effect occurs only for some kind of initial states. In other words, the phenomenon is sensitive to the initial conditions. One can expect the resurrection of the original entanglement to occur in a periodic way following each sudden death event. The length of the time interval for the zero entanglement is dependent on not only the degree of entanglement of the initial state but also the initial state. And the influence of dipole-dipole interaction and different atomic initial states on entanglement between atoms are discussed. The sudden death effect can be weakened by the introducing of dipole-dipole interaction.

Key words: Concurrence; Two photons Tavis-Cummings model; Sudden death effect of entanglement; Dipole-dipole interaction
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Entanglement is one of the most striking features of quantum mechanics. It plays a fundamental role in almost all efficient protocols of quantum computation (QC) and quantum information processing (QIP). In recent years, there has been an ongoing effort to characterize qualitatively and quantitatively the entanglement properties and apply then in quantum communication and information. Many schemes are proposed for generation of two or many particles entanglement. Among these, the system of atoms interacting with

* Corresponding author.
Email address: gf1978zhang@163.com (Guo-feng Zhang).

cavity field is a prominent scheme. Bose *et al.* show that entanglement can always arise in the interaction of an arbitrarily large system in any mixed state with a single qubit in a pure state[1,2]. They demonstrate this feature using the Jaynes-Cummings interaction of a two-level atom in a pure state with a field in a thermal state at an arbitrarily high temperature. Their method can be also used to study the entanglement between atom and cavity field in a loss Jaynes-Cummings model (JCM) and between two identical atoms in Tavis-Cummings model (TCM) [3]. Tessier *et al.* made a generalization of the JCM work to TCMS [4]. Boukobza *et al.* studied the relation between entanglement and entropy in JCM [5]. Recently, the dynamics of entanglement in bipartite systems has afresh received great attention since the work of Yu and Eberly [6], in which the entanglement between the two particles coupled with two independent environments became completely vanishing in a finite time. This surprising phenomenon, contrary to intuition based on experience about qubit decoherence, intrigues great interests [7,8,9,10,11,12]. In Ref.[8,9], the authors show that for a special initial state the entanglement disappeared in a finite time and then revived after a dark period because of the interaction between the particles, which is different from the works by Yu and Eberly [6] since in Ref.[8,9] the effects of interaction between the particles and the couplings to the same environment have been discussed extensively. In Ref.[3], the dipole-dipole interaction is ignored and only the case that when two atoms are initially in separate state is studied. What will happen if the atoms are initially in a entangled state and the dipole-dipole interaction is turned on. Thus, the purpose of this paper is to examine the entanglement character between atoms in the non-degenerate two photons TCM (TPTCM) and investigate the sudden death effect when the dipole-dipole interaction is turned off. The entanglement evolvement for a general case when considering the dipole-dipole interaction is given.

Consider the TPTCM where two identical two-level atoms are in a two-mode cavity field. By tracing cavity modes we are forcibly creating a two-qubit scenario, and various measures of entanglement are available. For a pair of qubits, all of them are equivalent, in the sense that when any one of them indicates no entanglement (separable states), the others also indicate no entanglement. Throughout the paper we will use Wootters's [13] concurrence $C(\rho)$ as the conveniently normalized entanglement measure $(1 \geq C \geq 0)$.

The dipole-dipole interaction of the atoms can not be neglected when the distance of the atoms is less than the wave length in the cavity. The Hamiltonian can be given

$$\begin{aligned} H = \sum_{i=a}^{b} \omega_i a_i^\dagger a_i + \frac{1}{2}\omega_0 \sum_{l=A}^{B} \sigma_l^z + g \sum_{l=A}^{B} (a_a^\dagger a_b^\dagger \sigma_l^- + a_a a_b \sigma_l^+) \\ + \Omega(\sigma_A^+ \sigma_B^- + \sigma_B^+ \sigma_A^-), \end{aligned} \tag{1}$$

where a_i and $a_i^\dagger$ $(i = a, b)$ denote the annihilation and creation operators of the frequency ω_i quantization field. ω_0 is the atomic Rabi frequency. $\sigma_z^l = |e\rangle_{ll}\langle e| - |g\rangle_{ll}\langle g|$, $\sigma_+^l = |e\rangle_{ll}\langle g|$, $\sigma_-^l = |g\rangle_{ll}\langle e|$ are the atomic operators with $|e\rangle_l$ and $|g\rangle_l$ being the excited and ground states of the lth atom $(l = A, B)$. g is the coupling constant between atom and field, Ω is dipole-dipole coupling strength between atoms.

For greatest simplicity, we assume that both cavities modes are prepared initially in the vacuum state $|0_a\rangle \otimes |0_b\rangle = |00\rangle$ and the two atoms are in a pure entangled state specified below. This allows a uniform measure of quantum entanglement—concurrence. In principle, there are six different concurrences that provide information about the overall entanglements that may arise. With an obvious notation we can denote these as C^{Aa}, C^{Ab}, C^{Ba}, C^{Bb}, C^{AB}, C^{ab}. Here we confine our attention to C^{AB}.

We assume that the two atoms are initially in a partially entangled atomic pure state that is a combination of Bell states usually denoted $|\Psi^{\pm}\rangle$, we have

$$|\Psi_{atom}(0)\rangle = \cos[\alpha]|eg\rangle + \sin[\alpha]|ge\rangle, \tag{2}$$

with the first index denoting the state of atom A and the second denoting the state of atom B. Thus the initial state for the system (1) can be given by

$$|\Psi(0)\rangle = |\Psi_{atom}(0)\rangle \otimes |00\rangle = \cos[\alpha]|eg00\rangle + \sin[\alpha]|ge00\rangle. \tag{3}$$

The solution of the system in terms of the standard basis can be written as

$$|\Psi(t)\rangle = x_1|eg00\rangle + x_2|ge00\rangle + x_3|gg11\rangle, \tag{4}$$

where $\omega_0 = \omega_a + \omega_b$ is assumed. The corresponding coefficients is given by

$$\begin{aligned}
x_1 &= \frac{\Lambda}{4}[\theta_+(L_+e^{i\kappa T} - L_-) + 2\Xi\theta_-],\\
x_2 &= \frac{\Lambda}{4}[\theta_+(L_+e^{i\kappa T} - L_-) - 2\Xi\theta_-],\\
x_3 &= \frac{\Lambda\theta_+}{\kappa}(1 - e^{i\kappa T}).
\end{aligned} \tag{5}$$

with $\theta_{\pm} = \cos[\alpha] \pm \sin[\alpha]$ and $T = gt$. $\Lambda = e^{-i\kappa L_+ T/2}$, $\Xi = e^{i(3L_+ - 2)\kappa T/2}$, $L_{\pm} = \frac{\epsilon}{\kappa} \pm 1$, $\epsilon = \frac{\Omega}{g}$ and $\kappa = \sqrt{8 + \epsilon^2}$. The information about the entanglement of two atoms is contained in the reduced density matrix ρ^{AB} which can be obtained from (4) by tracing out the photonic part of the total pure state. In

the standard basis $\{|1,1\rangle, |1,0\rangle, |0,1\rangle, |0,0\rangle\}$, the density matrix ρ^{AB} can be expressed as

$$\rho^{AB} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |x_1|^2 & x_1 x_2^* & 0 \\ 0 & x_2 x_1^* & |x_2|^2 & 0 \\ 0 & 0 & 0 & |x_3|^2 \end{pmatrix}, \tag{6}$$

it can be shown that the concurrence associated with the density matrix is given by

$$C^{AB}(t) = 2\max\{|x_1 x_2^*|, 0\}. \tag{7}$$

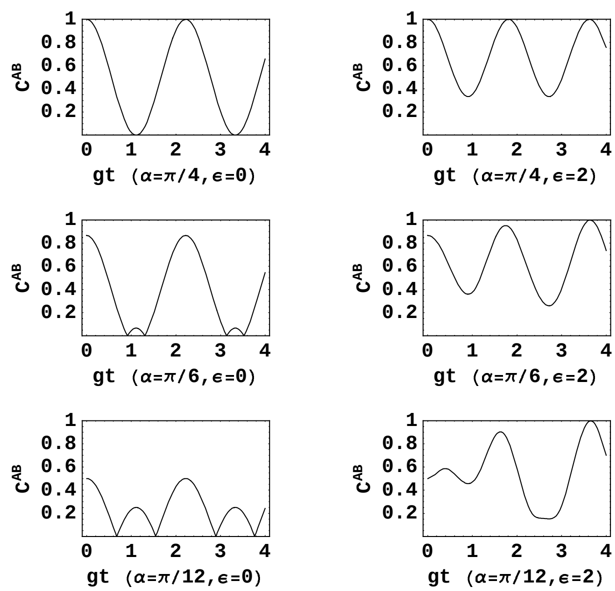

Fig. 1. The concurrence for atom-atom entanglement with the initial atomic state $|\Psi_{atom}(0)\rangle = \cos[\alpha]|eg\rangle + \sin[\alpha]|ge\rangle$.

Fig.1 shows the entanglement evolvement for the different α when $\epsilon = 0$ and $\epsilon = 2$. It is seen that entanglement can be strengthened by introducing the dipole-dipole interaction. Although Eq. (4) is not always an entangled state for atoms, with the increasing ϵ the coefficient of unpolarized state x_3 will be smaller until to be zero, thus the system state $|\Psi(t)\rangle \rightarrow x_1|eg00\rangle + x_2|ge00\rangle$ which is entangled for the two atoms. We can also see that the evolvement period gets shorter with the increasing ϵ. This is very easily understood since the period is governed by $\cos[\sqrt{8+\epsilon^2}gt]$. From the figures, we can know that dipole-dipole interaction can cause the entanglement be maximum although the initial entanglement may be very weak. The initial entanglement is determined by Eq.(2), $C(0) = 2\cos[\alpha]\sin[\alpha] = \sin[2\alpha]$ which is a fixed value.

$\epsilon = 0$, $C(t) = \frac{1}{4}(-1 + 3\sin[2\alpha] + \cos[\sqrt{8}gt](1 + \sin[2\alpha]))$, $C(t)_{max} = C(0)$. But for $\epsilon \neq 0$, $C(t)$ is dependent on ϵ, and it can arrive at the maximum value 1 for any ϵ and α at some evolvement time points which depend on ϵ and are independent of α. These can be seen from the right panel of Fig.1. In Eq.(5), x_1 , x_2 can arrive $\sqrt{2}/2$ and x_2 can be zero at the same time, for that case the entanglement is 1. This can be also understood from the physics since the independence of atoms will be weakened by the dipole-dipole interaction, thus the initial entangled state will be more entangled.

Now we assume that the initial state for the total system is in a combination of the other two Bell states $|\Phi^{\pm}\rangle$

$$|\Phi(0)\rangle = \cos[\alpha]|ee00\rangle + \sin[\alpha]|gg00\rangle, \tag{8}$$

in which case the state of the total system at time t can be expressed in the standard basis

$$|\Phi(t)\rangle = x_1|ee00\rangle + x_2|gg00\rangle + x_3|ge11\rangle + x_4|eg11\rangle + x_5|gg22\rangle, \tag{9}$$

where the coefficients are now given by

$$\begin{aligned}
x_1 =& \frac{\Gamma}{4}(\frac{\epsilon}{\eta}M_- + M_+ + 2e^{i(\epsilon+\eta)T/2}),\\
x_2 =& e^{i\lambda T}\sin[\alpha],\\
x_3 =& x_4 = \frac{\Gamma M_-}{\eta},\\
x_5 =& \frac{\Gamma}{4}(\frac{\epsilon}{\eta}M_- + M_+ - 2e^{i(\epsilon+\eta)T/2}).
\end{aligned} \tag{10}$$

with $\lambda = \frac{\omega}{g}$, $\eta = \sqrt{16 + \epsilon^2}$, $\Gamma = \cos[\alpha]e^{-i(2\lambda+\epsilon+\eta)T/2}$ and $M_{\pm} = 1 \pm e^{i\eta T}$. In the basis of $|ee\rangle$, $|eg\rangle$, $|ge\rangle$ and $|gg\rangle$ the reduced density matrix ρ^{AB} is now found to be

$$\rho^{AB} = \begin{pmatrix} |x_1|^2 & 0 & 0 & x_1x_2^* \\ 0 & |x_4|^2 & x_4x_3^* & 0 \\ 0 & x_3x_4^* & |x_3|^2 & 0 \\ x_2x_1^* & 0 & 0 & |x_2|^2 + |x_5|^2 \end{pmatrix}. \tag{11}$$

And the corresponding concurrence for this matrix can be obtained based on Ref.[13]. The numerical results are plotted in Fig.2. Unlike the previous case, figure 2 show that entanglement can fall abruptly to zero (the two lower curves

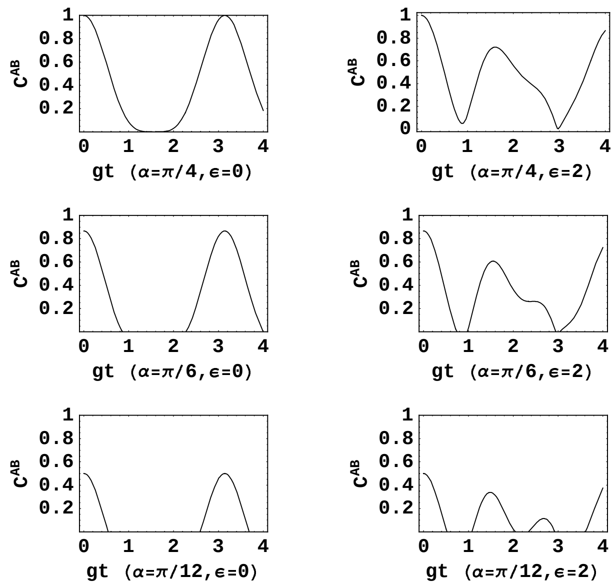

Fig. 2. The concurrence for atom-atom entanglement with the initial atomic state $|\Psi_{atom}(0)\rangle = \cos[\alpha]|ee\rangle + \sin[\alpha]|gg\rangle$.

in the figure), and it will remain to be zero for a period of time before entanglement recovers. The length of the time interval for the zero entanglement is dependent on the degree of entanglement of the initial state. The smaller the initial degree of entanglement is, the longer the state will stay in the disentangled separable state. Compared the right panel with the left one, one will find that the dipole-dipole interaction can cause the length of the time interval for the zero entanglement be shorter. When $\epsilon = 0$, let us first review the Hamiltonian Eq. (1), obviously the dynamics of the two independent two-level atoms is determined by the interaction terms in Eq. (1), which is the charge of the energy transfer between the system and field. We think it may be the energy transfer that leads to the entanglement sudden death (ESD). Originally ESD comes from the cutoff in the definition of concurrence. In our own points the main obstacle of explaining these phenomena is that it is unclear what the physical meanings of the concurrence are. Recently a great deal of works are denoted to the understanding from the energy of the system [14]. More recently a physical interpretation of concurrence for the bipartite systems has been provided based on the Casimir operator in [15]. It maybe open another door to understand concurrence as a physical quantity.

In summary, we have investigated the entanglement character existed in a non-degenerate two photons Tavis-Cummings model. We found that for the two different initial state of atoms, the entanglement evolvement appear dramatic difference. The length of the time interval for the zero entanglement is dependent on not only the degree of entanglement of the initial state but also the initial state. And the introducing of dipole-dipole interaction can cause the entanglement to be higher and weaken the difference of entanglement

evolvement between two kind different initial state of atoms.

Acknowledgements

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